Math 251 – Test 1 Overview (Section 1.7 – 2.6)

1.7: Introduction to continuity

- Graphical idea. (vertical asymptote, hole, break → Discontinuous)
- We sometimes say a function is <u>continuous at a point</u>, <u>continuous on an interval</u>, or if a function is continuous at every point, we say it is a "<u>continuous function.</u>"
- Intermediate value theorem. Informally, this theorem says that continuous functions cannot jump over *y*-values. For example: If f(*x*) is <u>continuous</u> on [1,2] and f(1)=6 and f(2)=7, then f(*a*)=6.5 for some value of *a* on [1, 2].
- If *f*(*x*) and *g*(*x*) are continuous at *x*=*a*, then so are (*f*+*g*)(x), (*fg*)(x). (*f/g*)(*x*) is continuous at *a* if *g*(*a*) is not 0, and if *f*(*g*(*x*)) is defined on an interval, then if *f*(*g*(*x*)) is continuous on that interval.

1.8: Limits

Limits as *x* → *c* (Book ex. Sin(x)/x →1 as x→0

- One-sided & Two-sided limits
- When limits do not exist:
  - Left and Right Limits don't agree (e.g. Step function)
  - Vertical asymptote at *c*
  - Wild oscillation near *c*. (e.g. lim x→0 sin(1/x) DNE)
- Limits at infinity (horizontal asymptote)
- Properties of limits. For example $\lim_{x\to a}(f(x)+g(x)) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$.
- More on continuity:
- *f*(*x*) is continuous at *c* means that the limit exists at *c* and is equal to *f*(*c*).

2.1 How Do We Measure Speed?

Understand the basic concept of a limit and be able to take simple limits.

a. $\lim_{x\to a} f(x) = L$ means that $f(x) \to L$ as $x \to a$.

Understand and be able to make connections between various interpretations of average velocity and instantaneous velocity.

a. Average Velocity = $\dfrac{\text{change in distance}}{\text{change in time}} = \dfrac{s(b)-s(a)}{b-a} = \dfrac{s(a+h)-s(a)}{h}$

b. Graphically, Average velocity is the slope of the line segment connecting the points $(a, s(a))$ and $((a+h), s(a+h))$

c. Instantaneous Velocity = $\lim_{h\to 0}\dfrac{s(a+h)-s(a)}{h} = s'(a)$

d. Graphically, Instantaneous velocity is the slope of the curve when *x*=*a* (i.e. the slope of the line tangent to the curve where *x*=*a*).

2.2 The Derivative at a Point

Understand and be able to make connections between various interpretations of average rate of change of a function over an interval, and the instantaneous rate of change at a point (i.e. the derivative at that point).

a. Avg. rate of change= $\dfrac{\text{change in function}}{\text{change in variable}} = \dfrac{f(b)-f(a)}{b-a} = \dfrac{f(a+h)-f(a)}{h}$

b. Graphically, Average rate of change is the slope of the line segment connecting the points $(a, f(a))$ and $((a+h), f(a+h))$

c. Instantaneous rate of change = $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h} = f'(a)$

d. Graphically, Instantaneous rate of change at *a* (the derivative at *a*) is the slope of the curve when *x=a* (i.e. the slope of the line tangent to the curve where *x=a*).

Be able to *estimate* the derivative of a function at a point graphically (by finding the slope of the curve) and by calculating the average rate of change over small intervals and seeing what number they approach.

2.3 Where *f* is a function, we define its *derivative function f'* by

$f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

A derivative function *f'* tells us the following information about the original function *f*:

If *f'* > 0 on an interval, then *f* is increasing on that interval.
If *f'* < 0 on an interval, then *f* if decreasing on that interval.

Given the graph of a function *f*, you should be able to sketch a graph of *f'*.

Be able use *and prove* with limits (no derivative shortcuts!) the following facts:

The derivative of a constant function: If *f(x)=k,* then *f'(x)=0*
The derivative of a linear function: If *f(x)=mx+b*, then *f'(x)=m.*
Be able to find other simple derivatives using limits.

2.4 Interpretations of the derivative

Given a function f that relates to a word problem, you should be able to interpret the *meaning* of its derivative f'. Here are some things that can help:

a. Sketch a rough graph of the original function f, and label the axes appropriately. This might give you an idea of where f' is positive or negative, or where f' is increasing or decreasing most quickly.

b. It's useful to be able to determine the unit of $f'(x)$.

The unit of $f'(x) = \dfrac{\text{Unit of } f}{\text{Unit of } x}$.

On a graph, the units of $f' = \dfrac{\text{Unit of the vertical axis}}{\text{Unit of the horizontal axis}}$.

You should be familiar with *leibnitz* ($\frac{dy}{dx}$) notation.

a. if $y=f(x)$, then $\frac{dy}{dx}$ means the same thing as $f'(x)$.

b. $\frac{d}{dx}f(x)$ means the same thing as $(f(x))'$

2.5 The second derivative

f' is the rate of change of f. So, f'' is the rate of change of the rate of change of f.

- If $f'' > 0$ on an interval, then f' is *increasing* on that interval.
  Therefore, f is *concave up* on that interval.
- If $f'' < 0$ on an interval, then f' is *decreasing* on that interval.
  Therefore, f is *concave down* on that interval.

If *s(t)* is a position function, then

$$s'(t) = \frac{\text{change in position}}{\text{change in time}} = \text{Velocity at time } t.$$

$$s''(t) = \frac{\text{change in velocity}}{\text{change in time}} = \text{Acceleration}$$

Leibniz notation for 2nd derivatives: $\frac{d^2y}{dx^2}$ means the same as $f''(x)$

2.6 Differentiability

f is not differentiable at points where:

a. The graph is not continuous
b. The graph has a sharp corner or cusp
c. The graph has a vertical tangent line

Theorem: If f is differentiable at a point, then it is continuous there. (However, just because a function is continuous at a point, doesn't mean it's differentiable there. For example, $y = |x|$ is continuous, but not differentiable at $x = 0$).